REDUCING THE EFFECTS OF RANDOM ERRORS IN SIX-PORT NETWORK ANALYSERS.

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ABSTRACT.

The performance of a Six-Port network analyser is degraded by the presence of noise and random error mechanisms.

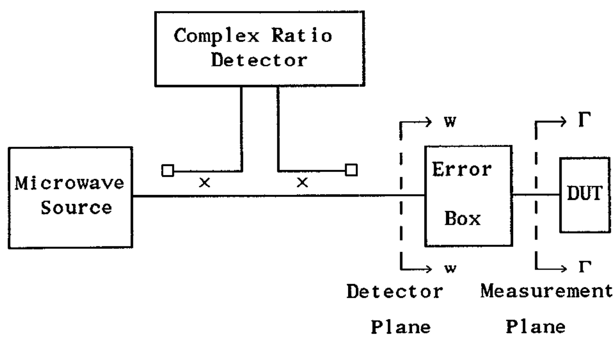
Experimental results demonstrate that the effects of random errors may be significantly reduced (10×) by employing a non-linear Kalman filter.

I. INTRODUCTION.

Control Engineers have exploited the Kalman filter to model noisy environments, however the benefits of this technique have not been recognized by the microwave community. An attractive property of the non-linear Kalman filter is the ability to deduce minimum variance estimates for a given parameter(s) from a collection of noisy observations. In the Six-Port environment this property is exploited, reducing the degrading effects of random errors that undermine the integrity of calibration and measurement.

II. KALMAN FILTER APPLICATION TO SIX-PORT CALIBRATION.

Fig 1. IDEAL 4 PORT AND ERROR BOX.



The Six-Port reflectometer may be modelled[1] as an ideal four-Port complex ratio detector in conjunction with a two-port error box, fig. 1. The physical properties of the ideal four-port complex ratio detector are related to the power detectors and complex detector wave ratio, w, by eqns. 1a-c.

$$\frac{P_1}{P_{ref}} = |w|^2 \qquad (1a)$$

$$\frac{P_2}{P_{ref}} = \frac{|w - w_1|^2}{\zeta} \qquad (1b)$$

$$\frac{P_3}{P_{ref}} = \frac{|w - w_2|^2}{\rho} \qquad (1c)$$

Calibration characterizes the instrumental parameters w_1, w_2, ζ and ρ of the network analyser. Engen[1] has shown that the complex detector wave ratio, w, may be eliminated from eqns 1a-c to form the power equation ;-

$$a\left[\frac{P_1}{P_{ref}}\right]^2 + b\zeta^2\left[\frac{P_2}{P_{ref}}\right]^2 + c\rho^2\left[\frac{P_3}{P_{ref}}\right]^2 + (c-a-b)\zeta\left[\frac{P_1P_2}{P_{ref}^2}\right]$$

$$+ (b-a-c)\rho\left[\frac{P_1P_3}{P_{ref}^2}\right] + (a-b-c)\zeta\rho\left[\frac{P_2P_3}{P_{ref}^2}\right] + a(a-b-c)\frac{P_1}{P_{ref}}$$

$$+ b(b-a-c)\zeta\frac{P_2}{P_{ref}} + c(c-a-b)\rho\frac{P_3}{P_{ref}} + abc = 0 \qquad (2a)$$

where

$$a = |w_1 - w_2|^2 \qquad (2b)$$

$$b = |w_2|^2 \qquad (2c)$$

$$c = |w_1|^2 \qquad (2d)$$

The Thru-Reflect-Line[2] and Sliding Short[1] are popular calibration techniques that both employ eqn. 2, which must be satisfied for all load conditions. Ordinarily this imposes no technical challenge; nine load conditions allow the instrumental parameters w_1, w_2, ζ and ρ to be readily computed.

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Unfortunately, non-linear algebraic manipulation of noisy power observations causes bias in the calculated estimates of the instrumental parameters[6]. Bias reduces measurement accuracy and may be eliminated by employing the non-linear Kalman filter to seek unbiased minimum variance estimates for the instrumental parameters.

The non-linear Kalman filter algorithm described by Gelb[3] has been extended to the Six-Port environment by Wright[4] and may be implemented directly. A simplified overview of filter operation is that the elements of the state vector $\underline{Xk}$ are optimized so that the error vector $\underline{Vk}$ is minimised for all load conditions. For the Six-Port environment;-

$$\underline{Xk} = \begin{bmatrix} w_1 \\ w_2 \\ \zeta \\ \rho \end{bmatrix} \qquad \underline{Vk} = \underset{\substack{\uparrow \\ \text{Ideal} \\ \text{Response}}}{0} - F(\underset{\substack{\uparrow \\ \text{Noisy Power} \\ \text{Observations}}}{P_1, P_2, P_3, Pref}:\ \underset{\substack{\uparrow \\ \text{Instrumental} \\ \text{Parameters}}}{w_1, w_2, \zeta, \rho})$$

where

$$F(P_1, P_2, P_3, Pref:\ w_1, w_2, \zeta, \rho) = \text{POWER EQN. (eqn. 2)}$$

The error vector, $\underline{Vk}$, comprises of the difference between the ideal response and actual value of the power equation. The filter operates over nine sets of power observations which correspond to the applied nine load conditions used in the standard calibration techniques[1 2]. If the non-linear Kalman filter seeks the correct minimum of the error vector, then the state vector will yield unbiased minimum variance estimates for the instrumental parameters.

III. EXPERIMENTAL RESULTS.

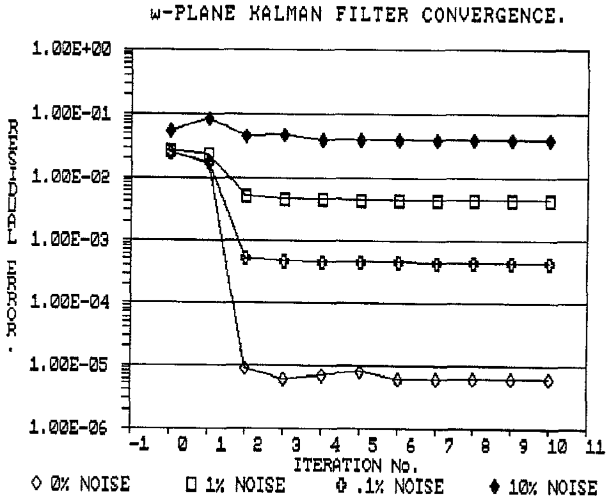
Experimental and computer simulations were conducted to ascertain the performance of the filter. A computer model of the Six-Port environment was subjected to Gaussian noise (variance = 1% of the average reference power level). Table 1 documents the estimates of the instrumental parameters as the filter iterates. The simulation confirms that the filter yields estimates for the instrumental parameters which converge towards the true values.

TABLE 1

Iteration Number	w_1 (instrumental parameters)	w_2	ζ	ρ	Error Residual
0	0.9626	1.3860	1.0455	1.5614	0.7143
1	0.8762	1.2885	0.9803	1.4690	0.5928
2	0.8664	1.2874	0.9529	1.4648	0.0120
3	0.8657	1.2873	0.9513	1.4645	0.0052
4	0.8657	1.2873	0.9513	1.4645	0.0047
5	0.8657	1.2873	0.9513	1.4645	0.0046
True values	0.8658	1.2875	0.9540	1.4645	0.0000

Fig. 2 demonstrates the fall in residual error for four noise levels and indicates that the filter rapidly reaches its noise floor. As the level of noise increases the uncertainty associated with each instrumental parameter increases. This is reflected by the increasing residual error.

Fig. 2.



The filter was also employed in an experimental Six-Port reflectometer operating at 2GHz. Table 2 charts the fall in error residual and corresponding improvement in instrumental parameters.

TABLE 2

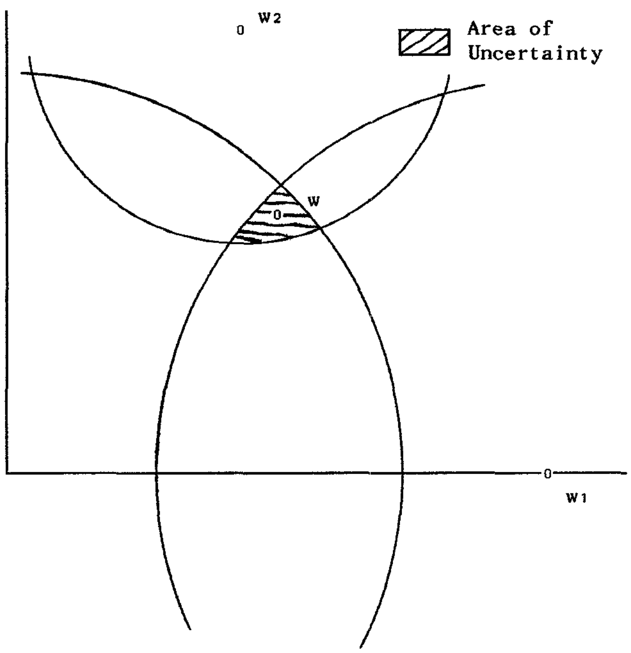
Iteration Number	w_1 (instrumental parameters)	w_2	ζ	ρ	Error Residual
0	1.3181	1.2543	0.1238	0.0745	0.1857
1	1.3178	1.2540	0.1240	0.0744	0.1193
5	1.3154	1.2528	0.1229	0.0747	0.1086
8	1.3122	1.2492	0.1225	0.0754	0.0404
11	1.3146	1.2507	0.1235	0.0758	0.0184
12	1.3148	1.2508	0.1237	0.0758	0.0183

When the error residual has reached a minimum the corresponding instrumental parameters are the unbiased minimum variance estimates. The experimental test indicates a significant decrease (10×) in error residual.

IV. KALMAN FILTER APPLICATION TO SIX-PORT MEASUREMENT.

Noise and random errors also effect the measurement of a device under test. Fig. 3 illustrates the problem.

Fig 3. COMPLEX w MEASUREMENT PLANE.



Rather than a unique intersection defining w, an area of uncertainty is produced of which the radial centre is often used to estimate w. Even though the minimum variance estimates for the instrumental parameters found during calibration are employed, which reduces the area of uncertainty, this estimate of w still suffers bias due to the observed noisy power readings. Eqns. 1 can now be recast to reveal the error associated with each power detector.

$$e_1 = \frac{P_1}{P_{ref}} - |w|^2 \qquad (3a)$$

$$e_2 = \frac{P_2}{P_{ref}} - \frac{|w - w_1|^2}{\xi} \qquad (3b)$$

$$e_3 = \frac{P_3}{P_{ref}} - \frac{|w - w_2|^2}{\rho} \qquad (3c)$$

Engen[5] has defined a least squares formulation that seeks a maximum likelihood estimate for the complex detector wave ratio, w, by minimising the error function, F(error).

$$F(error) = \frac{e_1^2}{\sigma_1^2} + \frac{e_2^2}{\sigma_2^2} + \frac{e_3^2}{\sigma_3^2} \qquad (4)$$

The variances, σ_i^2, are weights and proportional to the magnitude of the corresponding power detector reading. Thus eqn. 4 becomes the sum of the squares of the fractional errors in the power ratios. The requirement to assume a given variance for each power detector may be eliminated by employing the non-linear Kalman filter to process multiple observations of the power ratios; (in practice ten observations of P_i/P_{ref} proves sufficient). Direct application of the non-linear Kalman filter algorithm to the complex w-plane measurement defines the state vector, $\underline{Xk}$, and error vector, $\underline{Vk}$, as,

$$\underline{Xk} = \left[w \right] \qquad \underline{Vk} = \sum_{i=1}^{10} (e_{1i}^2 + e_{2i}^2 + e_{3i}^2) \qquad (5)$$

However the error vector, $\underline{Vk}$, violates the requirement to exhibit a zero mean. This criterion is essential to ensure a minimum variance estimate for the complex detector wave ratio, w, but may be overcome by redefining the error vector.

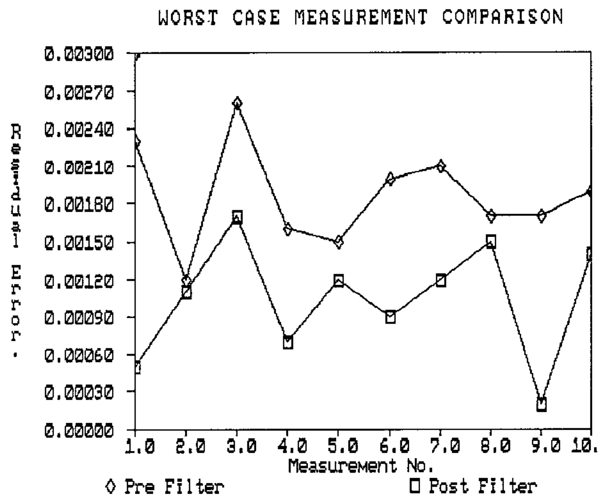
$$\underline{Vk} = \underbrace{\sum_{i=1}^{10}(e_{1i}^2 + e_{2i}^2 + e_{3i}^2)}_{\text{Previous Estimate}} - \underbrace{\sum_{i=1}^{10}(e_{1i}^2 + e_{2i}^2 + e_{3i}^2)}_{\text{Current Estimate}}$$

This causes the non-linear Kalman filter to seek stationary points rather than minimum points of the error vector. Fortunately this imposes few technical problems since the desired minimum is well defined and close to the radial centre, which provides an initial estimate for the complex detector wave ratio, w. Engen[5] has examined the error function given by eqn. 4 and concluded that secondary maxima and minima are at remote locations.

V. EXPERIMENTAL RESULTS.

Ten offset short circuits were measured with a computer simulation of a Six-Port network analyser operating at 2GHz and subject to 1% noise. Figure 4 compares the reflection coefficient error pre and post filter operation. In both cases the unbiased instrumental parameters (w_1, w_2, ζ, ρ) are employed.

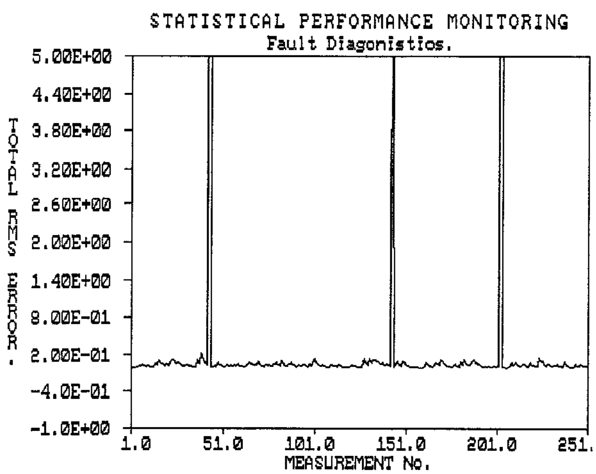
Fig 4.



Although error reduction is small, typically by a factor 2×–3×, this does represent a "fine adjustment" when compared to the error reduction in the calibration process.

In addition, monitoring the error residual, $\underline{V}k$, allows early identification of fault conditions arising in the Six-Port. Figure 5 illustrates a running average over 250 measurements, in which 3 power detector failures are deliberately induced. These faults are readily identified.

Fig. 5.



VI. CONCLUSIONS.

It has been demonstrated that the non-linear Kalman filter may be employed to significantly reduce the effect of random errors in the operation of a Six-Port Network Analyser.

VII. REFERENCES.

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